# Convergence in the Evolution of Nanodiamond Raman Spectra with Particle Size: A Theoretical Investigation 

Nanometer-sized diamond is an unusual form of carbon that has been observed in interstellar dust and meteorites. ${ }^{1,3}$ The detonation synthesis of nanodiamond (ND) was discovered in the 1960s, ${ }^{2}$ but nowadays, nanocrystalline diamond films are often grown by chemical vapor decomposition (CVD) methods from activated diluted hydrocarbon-hydrogen gas mixtures. ${ }^{4}$ Their physicochemical properties are hoped to become controllable by size- and shape-specific CVD synthesis, while already a variety of potential commercial ND applications were already reported during the past few years using the rather crude detonation synthesis. ${ }^{5-9}$ Due to their inherent biocompatibility, NDs have recently been "discovered" for medical applications, ${ }^{10}$ for instance, for drug delivery ${ }^{11,12}$ and biolabeling. ${ }^{13,14}$ In this context, use is made of surface or subsurface dopant species ${ }^{13,14}$ or surface functionalization. ${ }^{11,12}$

Structural characterization of NDs by means of spectroscopic techniques is of critical importance for the further development of the ND research field, as well as for practical applications. For this purpose, a thorough understanding of the factors affecting the spectroscopic properties is urgently required. Among the possible spectroscopic techniques, Raman vibrational spectroscopy has become one of the most valuable tools used for the structural characterization of NDs ${ }^{15-25}$ and their structural transformations at high temperatures. ${ }^{26} \mathrm{~A}$ number of characteristic features were proposed to discriminate NDs from other carbon nanomaterials. These features-Raman active bands that show up in spectra of nanocrystalline diamond in addition to the characteristic diamond signal (which is broadened and slightly red-shifted from its bulk value of $1332 \mathrm{~cm}^{-1}$ )—include the


#### Abstract

Structural characterization of nanodiamonds by vibrational spectroscopy requires knowledge of the factors determining the spectra. Raman spectroscopy is widely used to detect the diamond phase in nanodiamond powders and films, but several spectral features are still poorly understood. Here we present a theoretical study of the evolution of diamond hydrocarbon Raman spectra with increasing size, from the adamantane molecule to $\sim 3 \mathrm{~nm}$ large tetrahedral and octahedral particles of $T_{d}$ symmetry, containing up to about 1000 carbon atoms. The self-consistent-charge density functional tight-binding method (SCC-DFTB) was used for the calculation of harmonic first-order Raman spectra. We demonstrate very good agreement with Raman spectra computed by standard density functional theory (DFT) for the smaller model systems. The evolution of the Raman patterns is smooth, and convergence to the bulk limit could clearly be observed in case of the acoustic vibrational modes $\left(\omega_{\mathrm{A}}=0 \mathrm{~cm}^{-1}\right)$. We found a simple relationship between nanodiamond size and vibrational frequency, which is analogous to the corresponding equation for the radial breathing mode of single-walled carbon nanotubes. The $T_{2}$ modes of octahedral diamond hydrocarbons coalesce faster to the bulk optical vibrational mode (in experiment, $\omega_{0}=1332 \mathrm{~cm}^{-1}$ ) than those of tetrahedral particles, consistent with the fact that the bulk/ surface ratio is more favorable for octahedral particles. Our simulations unequivocally show that controversial Raman features around 500 and $1150 \mathrm{~cm}^{-1}$ do not originate from the nanodiamond crystals, and that the nanocrystal shape plays an important role in the appearance of the Raman spectra even in the 3 nm domain.


KEYWORDS: Raman intensities • vibrational spectroscopy • nanodiamonds • density functional tight binding $\cdot$ size evolution of Raman spectra
bands around 500, 600, 1150, 1400, 1450, and $1500-1800 \mathrm{~cm}^{-1} . .^{15-19,25}$ However, serious controversies exist preventing an unequivocal assignment of the observed vibrational bands to NDs, and it is far from clear whether these bands can be attributed to the NDs themselves. ${ }^{19-21}$ There is now a consensus that the peak around 1150 $\mathrm{cm}^{-1}$, which always appears together with the peak at $1450 \mathrm{~cm}^{-1}$ in poor quality CVDsynthesized NDs, originates from transpolyacetylene fragments. ${ }^{19,25}$ The origin of the double-resonant band at $1400 \mathrm{~cm}^{-1}$ and that of the broad asymmetric peak between 1500 and $1800 \mathrm{~cm}^{-1}$ has been ascribed to D - and G-bands of graphitic surface species, but in the case of the peak labeled as " G -band", it was recently shown that it is composed of $\mathrm{O}-\mathrm{H}$ bending with
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Figure 1. Molecular structures and formulas of the octahedral ( $\mathrm{A}, \mathrm{O} 2-\mathrm{O8}$ ) and tetrahedral ( $\mathrm{A}, \mathrm{T} 2-\mathrm{T} 10$ ) ND models considered in the present study.
contributions from $\mathrm{sp}^{2}$ carbon and $\mathrm{C}=\mathrm{O}$ stretching vibrations. ${ }^{25}$

Theoretical calculations of Raman spectra based on (scaled) harmonic vibrational frequencies are often performed for the assignment of experimental spectra. In the case of NDs, only two density functional theory
(DFT) and Hartree-Fock (HF)-based studies have been published previously, ${ }^{27,28}$ which reported contradictory conclusions regarding the peak assignments. Zhang and Zhang-using rather tiny molecular models of nanodiamonds up to the size of 44 carbon atoms—predicted ${ }^{27}$ that only the band at $\sim 500 \mathrm{~cm}^{-1}$
should be indicative of the presence of nanodiamonds, while the other experimental bands around 1140 and $1480 \mathrm{~cm}^{-1}$ were ascribed to Raman activity of $\mathrm{sp}^{2-}$ bonded carbon atoms. Filik et al.-using slightly larger models up to 88 carbon atoms-argued ${ }^{28}$ that actually none of these features originate from nanodiamonds, suggesting that the only Raman activity of NDs is produced by the broadened zone-center ( $1325 \mathrm{~cm}^{-1}$ ) mode and low-frequency ( $<100 \mathrm{~cm}^{-1}$ ) cage deformation modes. The authors conceded that their small model systems were not sufficiently large enough to make definite assignments of the experimental bands appearing in ND Raman spectra. A thorough theoretical study of the evolution of Raman spectra from adamantane to bulk diamond was considered impossible. ${ }^{27-29}$

The calculation of Raman spectra by standard ab initio or first-principles methods requires enormous computational resources, imposing severe restrictions on these conventional quantum chemical methods. Yet, the high density of diamond requires the use of model systems with a large number of atoms in even moderate-size atomistic model systems. We have shown in recent studies that the computationally much more economical self-consistent-charge density functional tight-binding (SCC-DFTB, abbreviated in the following text as DFTB) method ${ }^{30,31}$ is comparable in accuracy to conventional DFT methods for the theoretical investigation of vibrational IR and Raman spectra ${ }^{32,33}$ and can easily surpass the size limitations imposed on the first-principles methods. ${ }^{34-37}$ DFTB is therefore ideally suited for a study of the size and shape effect on the theoretical Raman spectra of finite-size atomistic model systems for ND species and in the position to shed light on the discrepancies between the previously published studies.

In this work, we focus on the size evolution of the Raman bands of nanocrystalline diamonds using two families (tetrahedral and octahedral) of finite molecular models. We selected these two families because they represent examples of least and higher sphericity of shape; NDs synthesized or found in nature certainly come in a great variety of shapes, most prominently quasi-spherical, but also irregular shapes with no symmetry at all. The simulated spectra can be expected to converge to the extremely simple Raman spectrum of bulk diamond, which consists of a single sharp peak at $1332 \mathrm{~cm}^{-1}$. We note that vibrational analysis based on harmonic force constant approximation is not expected to yield the fundamental vibrational frequencies. Moreover, the limitations in the accuracy of quantum chemical methods for predicting harmonic force constants can lead to deviations up to the order of $100 \mathrm{~cm}^{-1}$. The Raman spectra of nanocrystalline diamonds are, of course, more complex and depend strongly on the size and shape of the crystallite. In comparison with previous theoretical studies focusing on the simulation of

Raman spectra, significantly larger molecular models of NDs are used-up to $\sim 10^{3}$ carbon atoms-which enable us to visualize the Raman spectrum evolution during the transition from the molecular to the solid-state regime. It is also shown here that a large portion of the simulated Raman signal is generated by the atoms located in the vicinity of the outer surface of the clusters. An attempt to eliminate the surface effects by immobilizing the terminal atoms shows that the convergence of the Raman spectra toward that of the bulk diamond is remarkably enhanced by this procedure.

Model Systems. The model systems employed in the present study were constructed taking into account the following symmetry considerations. The symmetry space group of the diamond crystal is Fd 3 m , and the corresponding point group is $O_{h}$. Some of the symmetry elements of $O_{h}$, like inversion center, are present only in infinite diamond crystals. The highest symmetry point group appropriate for finite molecules is $T_{d}$. Under the restriction of $T_{d}$ symmetry, two different families of nanodiamond structures are conceivable: octahedral and tetrahedral. The building block used to construct these structures—adamantane $\left(\mathrm{C}_{10} \mathrm{H}_{16}\right)$ referred to in the following as A-is identical for both families. By replicating the carbon backbone of the building block an appropriate number of times, we obtain seven octahedral ( $\mathrm{O} 2-\mathrm{O}$ ) and nine tetrahedral (T2-T10) nanodiamond structures. Their molecular structures and chemical formulas are given in Figure 1. The outer surface of all of the models is saturated with hydrogen atoms. The largest considered structure (O8) consists of 969 carbon atoms. The size of the investigated models ranges from 0.3 nm (A) to 2.6 nm (T10) and 2.9 nm (O8) across the maximal carbon - carbon distance ( $r_{\text {max }}$ ). It is convenient to give here an equation allowing an estimate for the size of the model structures from the number of the carbon atoms and vice versa. It was obtained by fitting $r_{\text {max }}$ to the corresponding number of carbon atoms ( $n$ ) for the five largest structures of each family. The correlation coefficient was practically equal to unity. The effective formula is given by

$$
\begin{equation*}
r_{\max }=a n^{b}-c \tag{1}
\end{equation*}
$$

where $a=3.264 \AA, b=0.3349$, and $c=3.5 \AA$ for the octahedral models and $a=3.168 \AA, b=0.3498$, and $c=$ $4.5 \AA$ for the tetrahedral models. As expected, the size of the models scales approximately as a cubic root of $n$.

## RESULTS AND DISCUSSION

Comparison of DFTB with DFT and Previous Results. Both DFTB and DFT methods were used to compute Raman spectra of smaller model systems (A, T2-T4, O2, and O 3 ), while for larger models ( $\mathrm{T} 5-\mathrm{T} 10$ and $\mathrm{O} 4-\mathrm{O} 8$ ), only the DFTB method could be employed. To assess the quality of the DFTB Raman spectra, a comparison of the DFTB and DFT (BLYP/3-21G and BLYP/6-31+G*) Ra-


Figure 2. Comparison of the Raman spectra calculated using BLYP/3-21G, BLYP/6-31G*, and SCC-DFTB for the A, T2, T3, T4, O 2 , and O 3 models.
man spectra computed for the smaller models is given in Figure 2. The overall agreement is satisfactory, with the largest discrepancies concerning a somewhat altered intensity pattern and shifts (up to $50 \mathrm{~cm}^{-1}$ ) in the position of some of the peaks. This picture is consistent with our previous studies of the SCC-DFTB Raman spectra of organic molecules and fullerenes. ${ }^{32,34,38}$ In the present study, the main differences between the spectra are associated with the vibrational activity of the $\mathrm{CH}_{2}$ groups at the model system vertices, which, fortu-
nately, is irrelevant for the main purpose of our current investigation. It was earlier demonstrated by Filik et al. ${ }^{28}$ that a similar degree of change in the simulated Raman spectrum of adamantane can be induced upon the change of the computational method (HF vs DFT) and/or the basis set.

The previous theoretical studies of nanodiamond Raman spectra ${ }^{27,28}$ were limited to very small molecular models with less than 100 atoms. The largest structure studied by Zhang and Zhang ${ }^{27}\left(\mathrm{C}_{44} \mathrm{H}_{42}\right)$ is interme-
diate between our O 2 and O 3 models. Our simulated O 2 spectrum agrees reasonably well with the O 2 spectrum computed by these authors, who used a different level of DFT, namely, B3LYP/6-31G*. The largest atomistic models used by Filik et al. ${ }^{28}$ comprise approximately 90 carbon atoms and are directly related to our O3 and T4 models. Unfortunately, it is not feasible to compare their Raman spectra with ours because they used Raman activities to display their data instead of more appropriate Raman intensities.

General Evolution of Raman Spectra of Octahedral and
Tetrahedral NDs. Before presenting the results for the evolution of ND Raman spectra with increasing model system size, we shortly discuss the Raman spectrum of infinitely large diamond, which constitutes the bulk limit toward which our finite ND models are expected to converge. The phonon dispersion structure of diamond consists of six phonon bands: three acoustic and three optical. Their frequencies can be experimentally determined using first-order Raman and Brillouin scattering only in the vicinity of the $\Gamma$ point owing to the crystal momentum conservation. At $\Gamma$, the optical ( $\omega_{\mathrm{O}}=1332$ $\mathrm{cm}^{-1}$ ) and acoustic ( $\omega_{\mathrm{A}}=0 \mathrm{~cm}^{-1}$ ) phonons are degenerate. Therefore, the first-order Raman spectrum of bulk diamond consists only of a single sharp band located at $1332 \mathrm{~cm}^{-1} .{ }^{39-42}$

Clearly, the finite-size models considered here possess no translational symmetry and can be treated as very large molecules. Consequently, the Raman spectra of NDs can be expected to consist of a large number of bands corresponding to the Raman active vibrational modes. Their activities are, in principle, governed only by the $T_{d}$ point group selection rules. In practice, however, it is reasonable to expect that, for large finite models, the vibrational densities of states (VDOSs) will resemble and in the limit of an infinite number of atoms will converge to the phonon density of states of the crystal diamond. This expected convergence behavior can be expressed alternatively by saying that the vibrational frequencies of large models studied here probe uniformly the phonon dispersion surfaces of diamond over the whole Brillouin zone. Therefore, it is reasonable to expect that the Raman activities of vibrational modes not corresponding to the $\Gamma$ point (i.e., out-of-phase with respect to the relative motion of unit cells inside the finite models) will contribute only marginally to the simulated Raman spectra. For smaller models, where the similarity is only approximate, the optical phonons at $\Gamma$ would have a nonvanishing overlap with a larger number of vibrational modes of a finite model leading to the characteristic asymmetric line width broadening of the red-shifted zone-center mode and producing a number of additional Raman features. Note that this molecular interpretation is essentially equivalent to the phonon-confinement model ${ }^{43}$ that was used previously to interpret the Raman spectra of nanocrystalline diamonds. ${ }^{44}$ A similar interpretation can be given for
the acoustic phonons that manifest themselves in finite models as a comb of low-frequency bands usually referred to as the cage deformation modes.

Speaking more precisely, it would be reasonable to expect the aforementioned behavior if we could avoid including the terminal hydrogen atoms in our model systems. The C-H stretching modes ( $2850-3400 \mathrm{~cm}^{-1}$ ) are not interfering with the studied spectral region, but, unfortunately, the $\mathrm{H}-\mathrm{C}-\mathrm{H}$ bending modes ( $1350-1460 \mathrm{~cm}^{-1}$ ) with a priori unpredictable intensity pattern do complicate the analysis of the simulated spectra. This complication is solved by setting the atomic mass of hydrogen to a large value ( 10000 amu ), which corresponds effectively to shifting away the intervening features from the studied spectral window. The Raman spectra obtained using this modification will be further referred to as the "infinitely heavy hydrogens" (briefly, infH) Raman spectra.

With the preceding theoretical considerations in mind, we are now ready to analyze the results of our simulations. As expected, the Raman spectra obtained for the hydrogen-terminated tetrahedral and octahedral finite models (shown in Figure 3) display a large number of active bands. Most of these bands have very small intensity. The bands are grouped in two families, one with $\omega<400 \mathrm{~cm}^{-1}$ and one centered around $1200 \mathrm{~cm}^{-1}$. This pattern is more pronounced for larger NDs. The evolution of the simulated Raman signal with growing size of the models is very regular. The positions of the bands and their intensities change smoothly, demonstrating a clear convergence pattern underlying the Raman spectra of NDs. Two features of this evolution are worth particular attention. First, the number of intensive Raman bands is considerably reduced with the growing size of the model. For the largest model systems size of the tetrahedral family (T10), there are only seven intensive bands, while for that of the octahedral family (O8), almost all Raman intensity is carried by five bands. We use this regularity below to discuss the anticipated Raman spectra of much larger NDs, for which we could not afford explicit DFTB calculations. Second, the low-frequency bands experience a quite substantial red shift during the evolution, while the other bands (with $\omega>800 \mathrm{~cm}^{-1}$ ) remain almost unchanged in the same region of the spectra for different size model systems. This feature allows for tracking back the origin of the active Raman bands of NDs to those of adamantane, which are much easier to interpret. A detailed analysis along this line is given below.

Evolution of the Low-Frequency Bands. The low-frequency bands originate from the breathing and cagedeformation modes and clearly can be associated with the acoustic phonons of diamond. Their frequencies and intensities converge rapidly with growing model size. For the largest studied models, O 8 and T10, all of the low-frequency Raman bands are located below 200 $\mathrm{cm}^{-1}$. The intensity and position pattern of these bands


Figure 3. Evolution of the simulated Raman spectra for the tetrahedral and octahedral finite models of nanodiamonds.
is quite different for both studied families of structures. The octahedral models display a quartet of active bands, corresponding to the $E, T_{2}, T_{2}$, and $A$ irreps, of approximately similar intensity. During evolution, the spacing between the peaks decreases and their frequencies approach zero. The tetrahedral models display two doublets of active bands, each of them showing similar behavior as discussed for the quartet of octahedral models. The active modes correspond to the following irreducible representations (irreps): $E_{1} T_{2}$, $E, T_{2}$, and $A$; the second pair of the $E$ and $T_{2}$ modes coincide, appearing as a single peak. Despite of the fact that the pattern is different for both families, it is clear that in the bulk limit of the size evolution they converge toward the same Rayleigh line. The strong dependence of the position of these peaks on the size of the model can be used as a diagnostic for determination of the size of the nanocrystalline grains of diamond, as suggested earlier by Filik et al. ${ }^{28}$ We have fitted the po-
sitions of the observed bands to the following formula

$$
\begin{equation*}
\omega=a n^{-b} \tag{2}
\end{equation*}
$$

with $n$ being the number of atoms in a given structure, using the data for the five largest models of each family. The quality of the fit is almost perfect (for details, see Supporting Information). Since the exponent $b$ is roughly equal to $1 / 3$, the low-frequency modes scale approximately inversely with the size of the model $r_{\text {max }}$. This behavior agrees well with the previous experimental semiconductor ${ }^{45-47}$ and theoretical results. ${ }^{48-50}$ To allow the prediction of the position of the lowfrequency bands in general size nanocrystals, we give here the fitted coefficients for the limiting lowfrequency bands of each model. They are $a=882 \mathrm{~cm}^{-1}$ and $b=0.3691$ for the lowest band $(E)$ and $a=1410$ $\mathrm{cm}^{-1}$ and $b=0.3431$ for the highest band $(A)$ of the tetrahedral model, and $a=1005 \mathrm{~cm}^{-1}$ and $b=0.3034$ for the lowest band $(E)$ and $a=1578 \mathrm{~cm}^{-1}$ and $b=$ 0.3231 for the highest band $(A)$ of the octahedral model. Assuming that the regular pattern described here can be extrapolated also to much larger systems, eq 2 together with the given coefficients can be used to estimate the anticipated positions of the low-frequency bands in the recently reported Raman spectra of two single-crystal nanodiamonds. ${ }^{23,24}$ The nanodiamond of the size 90 nm , which contains approximately 20 [10.5] million carbon atoms and corresponds effectively to a O246 [T314] model system, would have the lowfrequency bands located in the range 5.7-6.9 [2.2-6.2] $\mathrm{cm}^{-1}$ assuming the octahedral [tetrahedral] regime. Analogously, the smaller nanodiamond of the size 35 nm, which contains approximately 1.2 [0.7] million carbon atoms and corresponds effectively to our O96 [T126] model, would have the low-frequency bands located in the range $14.0-17.2[6.0-14.8] \mathrm{cm}^{-1}$. It would be relatively easy to distinguish between the two regimes, as the octahedral models display almost equally spaced bands, while for the tetrahedral models, the separation between the doublets becomes even more pronounced for large NDs. We note that eq 2 can be written as an analogue of the dependence of the radial breathing mode of single-walled carbon nanotubes on the inverse tube diameter ${ }^{54}$

$$
\begin{equation*}
\omega=a n^{-b} \approx \frac{a^{\prime}}{r_{\max }} \tag{3}
\end{equation*}
$$

where $r_{\text {max }}$ takes the place of the tube diameter. We hope that these considerations can prove helpful for assessing the size and shape of diamond nanocrystals in low-frequency Raman spectroscopy experiments. We are aware that unfortunately, at present, the experimental sample quality for the size range of NDs, where these modes are detectable by Raman spectroscopy, does not allow detection of these features.

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Evolution of the $800-1500 \mathrm{~cm}^{-1}$ Spectral Window. For the octahedral models, the bands located between 800 and $1500 \mathrm{~cm}^{-1}$ converge toward a single, dominant peak. DFTB predicts the position of this $T_{2}$ band, corresponding to the optical phonons of diamond, at approximately $1200 \mathrm{~cm}^{-1}$. A detailed discussion of the evolution history for the $T_{2}$ band is given in the next section for both families of models. For the tetrahedral models, the convergence behavior is again quite different. The most important observation is that the models used by us are not sufficiently large to predict uniquely the Raman spectra of larger tetrahedral nanodiamonds. One may expect that the presented series of Raman spectra of T2-T10 would converge to three separated bands located around 1050, 1200, and 1450 $\mathrm{cm}^{-1}$. In analogy with the octahedral models, the signal at $1200 \mathrm{~cm}^{-1}$ corresponds to the zone-center $T_{2}$ band. The remaining two bands, at 1050 and $1450 \mathrm{~cm}^{-1}$, presumably fade away only for much larger numbers of atoms. Their origin can be easily traced back to specific vibrational modes of adamantane. The $E$ band at 1450 $\mathrm{cm}^{-1}$, whose position is practically constant for all models, originates from the $1453 \mathrm{~cm}^{-1}$ scissoring motion of the adamantane $\mathrm{CH}_{2}$ groups (observed in experiment $^{51}$ at $1439 \mathrm{~cm}^{-1}$ ). Clearly, there are very few reasons for this band to change its position during the evolution. Note that this band is present in all of the Raman spectra simulated by us, but for the octahedral models, it is rapidly shadowed by other, more intensive bands, while for the tetrahedral models, the decay of its relative intensity is much slower. The other mode, around $1050 \mathrm{~cm}^{-1}$, evolves from the $T_{2} 942 \mathrm{~cm}^{-1}$ vibration of adamantane (observed in experiment ${ }^{51}$ at 970 $\mathrm{cm}^{-1}$ ), which is a specific amalgam of $\mathrm{CH}_{2}$ rocking, CC stretching, and CCC deformation. For larger models, the constituent contributions change slightly and the mode shifts toward higher frequencies: $1025 \mathrm{~cm}^{-1}$ for T2, $1038 \mathrm{~cm}^{-1}$ for T3, and $1048 \mathrm{~cm}^{-1}$ for T4.

Evolution of the $T_{2}$ Raman Band. The rate of convergence of the simulated Raman spectra of NDs can be most conveniently inspected by the analysis of the spectral region around $1100-1300 \mathrm{~cm}^{-1}$, which effectively corresponds in DFTB to the optical phonons of diamond at $\Gamma$. The evolution of this region in the Raman spectra studied here of the three largest tetrahedral and octahedral structures is shown in Figure 4. For the octahedral structures, the convergence toward the diamond band is very clear and rapid. The intensity of the $T_{2}$ band is enhanced with the growing size of the model, while the intensity of the other bands is practically unaltered. The position of the $T_{2}$ band depends on the size of the model: $1185 \mathrm{~cm}^{-1}$ for O6, $1192 \mathrm{~cm}^{-1}$ for O7, and 1196 $\mathrm{cm}^{-1}$ for O8. (The corresponding diameters of these NDs are 2.2, 2.5, and 2.9 nm.) Unfortunately, our models are still too small to extrapolate this trend further as it has been done for the low-frequency modes. Such an extrapolation would allow us to evaluate how much
the displaced position of the $T_{2}$ band in our simulation ( $\sim 1200$ vs $1332 \mathrm{~cm}^{-1}$ in experiment) is caused by the harmonic approximation and the deficiencies of the DFTB method and how much it originates from the not fully converged size of our model. A certain estimation of this effect can be obtained by computing the DFTB vibrational frequencies for butane $\left(\mathrm{C}_{4} \mathrm{H}_{10}\right)$ and comparing the position of the symmetric $\mathrm{C} \leftrightarrow \mathrm{C}-\mathrm{C} \leftrightarrow \mathrm{C}$ stretch mode ( $982 \mathrm{~cm}^{-1}$ ) with its experimental value (1059 $\mathrm{cm}^{-1}$ ). ${ }^{52}$ The difference of approximately $80 \mathrm{~cm}^{-1}$ suggests that in the DFTB Raman spectrum performed for a fully converged model one may expect the position of the $T_{2}$ band at approximately $1250 \mathrm{~cm}^{-1}$. In this context, the red shift of the $T_{2}$ band by $50 \mathrm{~cm}^{-1}$ observed for small models is not surprising; an analogous effect was observed earlier in experiment ${ }^{23,24}$ and in theory ${ }^{17,44}$ when the size of the diamond nanocrystals was reduced from $\infty$ to 5.0 nm . We note in comparison that harmonic vibrational frequency calculations for bulk diamond using periodic boundary conditions predict the position of the $T_{2}$ anywhere between 1277 and 1361 $\mathrm{cm}^{-1.53}$

The evolution pattern for the tetrahedral structures is much more structured. As can be seen from Figure 4, the Raman signal in the studied spectral window is actually dominated by an $E$ band. The relative intensity of the $T_{2}$ band is slowly growing and may/will dominate the $E$ band for much larger models than those inspected in the present study. The position of the $T_{2}$ band is gradually blue-shifted with the growing size of the model, analogously as for the octahedral models, while the position of the $E$ band is practically uninfluenced.

Evolution of the Simulated infH Raman Spectra. It is easy to understand the difference between the Raman spectra of the tetrahedral and octahedral models taking into account the C/H ratio, closely related to the bulk/surface ratio for both the considered sets of models. The bulk/ surface ratio (for an object with average side length $R$, volume scales with $R^{3} \sim n$, and surface area scales with $\sim R^{2}$ ) naturally increases as a function of $n^{1 / 3}$ and linearly with the number of layers (which is directly proportional to $R$ ); however, for small model systems, deviations from ideal scaling are observed. Moreover, different polyhedra such as the octahedron and the tetrahedron possess different volume to surface ratios. This ratio is considerably smaller for the tetrahedral models, suggesting slower convergence of the corresponding Raman spectra patterns (for details, see Figures S1 and S2 in the Supporting Information). As mentioned earlier, a certain way to evade this problem is by artificially increasing the masses of the terminating hydrogen atoms in the mass-weighted Hessian matrix prior to diagonalization. Since the low-frequency modes are related to breathing and shearing, they are suppressed by this technique, and we are left only with the bands converging to the optical phonon of dia-

## Tetrahedral models



Octahedral models


Figure 4. Evolution of the $T_{2}$ band in the simulated Raman spectra of the three largest tetrahedral (T8-T10) and octahedral (O6-O8) models of nanodiamonds.
mond. Consequently, the simulated infH Raman spectra simulated using only the $T_{2}$ modes (shown in Figure 5) are markedly simpler and consist only of a single group of peaks corresponding to the optical phonon of diamond. The evolution history and the final Raman pattern are quite similar for both families of NDs. For smaller models, a number of moderately intense peaks are observed. When the size of the models grow, these peaks merge together developing a single, dominating band centered at approximately $1200 \mathrm{~cm}^{-1}$. The convergence toward the Raman spectrum of a diamond crystal is remarkably rapid. For the largest studied models, the only different spectral feature is a satellite band
consisting of low-intensity peaks between 1250 and $1300 \mathrm{~cm}^{-1}$. Clearly, as can be inferred from their evolution behavior, this band will vanish for larger finite models.

The details of the evolution history of the zonecenter $T_{2}$ band for the simulated infH Raman spectra is presented in Figure 6 for both families of models. As mentioned earlier, the convergence is rapid. The resultant band consists of a number of closely spaced Raman signals; their number is substantially larger for the tetrahedral models. The half-width of the effective envelope of the Raman band is reduced with the growing size of the model; this effect is particularly pronounced


Figure 5. Evolution of the simulated infH Raman spectra for the tetrahedral and octahedral models of nanodiamonds. For details, see text.
for the tetrahedral NDs. Note that this observation is in line with the earlier theoretical and experimental results obtained for large nanodiamond crystals, where the position of the $T_{2}$ band and its half-width were shown to be closely related to the size of the ND grains (for comparison, see in particular Figure 8 of ref 44 and Figure 3 of ref 17).

## SUMMARY AND CONCLUSIONS

The size evolution of the Raman spectra of octahedral and tetrahedral NDs with a diameter range of $0.3-2.9 \mathrm{~nm}$ has been studied using the DFT and DFTB methods. To the best of our knowledge, this is the first time that the size evolution of Raman spectra for molecules up to about 1000 atoms was followed using a quantum chemical method of similar accuracy to DFT. A clear and smooth convergence toward the Raman spectrum of a diamond crystal has been observed, par-
ticularly in the octahedral regime. The Raman spectrum of the largest studied octahedral structure, O8 $\left(\mathrm{C}_{969} \mathrm{H}_{324}\right)$, is dominated by a single $T_{2}$ band at 1200 $\mathrm{cm}^{-1}$ corresponding to the optical phonons of diamond at $\Gamma$, and a comb of low-frequency bands located between 120 and $170 \mathrm{~cm}^{-1}$, which correspond to the diamond's acoustic phonons. In the tetrahedral regime, two additional spectral features are detected, at 1050 and $1450 \mathrm{~cm}^{-1}$, which we assume will fade away for model systems larger than those studied here since they originate from surface effects. This assumption is based on the fact that these features do not persist in the octahedral regime and, therefore, are expected to similarly fade away since the Raman spectra for both polyhedra types have to converge in the bulk limit. The position of the $T_{2}$ band for both families of structures is gradually blue-shifted with the growing size of the

Octahedral models


Tetrahedral models


Figure 6. Formation of the $T_{2}$ band of diamond in the simulated infH Raman spectra of the octahedral and tetrahedral nanodiamonds.
model. This suggests that one of the reasons for the observed discrepancy between the positions of the simulated ( $\sim 1200 \mathrm{~cm}^{-1}$ ) and experimental ( 1325 to 1332 $\mathrm{cm}^{-1}$ ) band-in addition to that of the harmonic approximation and the inherently approximate nature of DFTB—may be the insufficiently large size of the models studied here. This observation particularly applies to the family of tetrahedral structures, for which the convergent trend in the evolution of the non-lowfrequency bands requires large model systems.

We obtained three important findings from this work. (i) We have given a quantitative relationship between the ND size and the position of the lowfrequency Raman active peaks (eqs 2 and 3 ) as a an analogue of the dependence of the frequency of the radial breathing mode (RBM) on the inverse diameter in single-walled carbon nanotubes. ${ }^{54}$ (ii) The Raman spec-
tra evolution pattern depends quite strongly on the shape (here octahedral versus tetrahedral family) of the crystalline ND grains. To the best of our knowledge, the influence of the shape on the appearance of the ND Raman spectra has not been considered before. (iii) No evidence of other spectral features previously suggested for Raman characterization of ND samples is found in our simulations, in good agreement with previous considerations from experiments. ${ }^{19,25}$ Our work confirms the theoretical predictions made by Filik et al. ${ }^{28}$ that the only Raman activity expected for NDs is associated with the zone-center mode around $1325 \mathrm{~cm}^{-1}$ and the low-frequency cage deformation and breathing modes. Studies of the effect of defects, different and irregular shapes, and surface functional groups on ND vibrational spectra are currently ongoing in our laboratories.

## METHODS

Density functional theory (DFT) calculations using BLYP/3-21G ${ }^{55-57}$ and BLYP/6-31+G*5,56,58-60 were performed using the implementation in the GAUSSIAN 03 quantum chemistry code with default convergence criteria. ${ }^{61}$ BLYP is known to yield harmonic vibrational frequencies that are in better agreement with experiment than hybrid functionals such as B3LYP, which inherit the systematic error of the Hartree-Fock method. We computed equilibrium geometries, harmonic vibrational frequencies, and Raman activities following the standard procedures. SCC-DFTB calculations were performed using a standalone FORTRAN-based DFTB code with an implemented analytical Hessian module. ${ }^{38,62,63}$ For geometric optimization of the nanodiamond (ND) models, we adopted the SCC energy convergence criterion of $10^{-12}$ hartree for energies and $10^{-6}$ for the
gradient in atomic units. The Hessian and computed harmonic frequencies were left unscaled to allow for transferability of the reported data. The Raman intensities were obtained ${ }^{38}$ from the computed activities assuming the following experimental conditions: $T=25^{\circ} \mathrm{C}$ and $\lambda_{\text {laser }}=514.5 \mathrm{~nm}$. The simulated DFT and DFTB Raman spectra were constructed by superposing Gaussians (with half-width of $1.5 \mathrm{~cm}^{-1}$ ) located at the computed harmonic vibrational frequencies with heights corresponding to the calculated intensity. The presented Raman spectra are given in arbitrary intensity units; the area under each spectrum is normalized to unity.

It was found that the hydrogen-terminated surface of the investigated models produces a large number of spurious Raman peaks. In order to minimize this undesirable effect, the masses of the terminal hydrogen atoms were set to a very large value $(10000 \mathrm{amu})$ in analogy with previous work on polycyclic aro-
matic hydrocarbons (PAH) ${ }^{64}$ and nanodiamonds. ${ }^{28}$ This change effectively eliminates the surface contribution to the Raman spectra, allowing only the inner carbons to vibrate.

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Supporting Information Available: PDF file giving additional details of our simulations (equilibrium geometries, vibrational frequencies and intensities, fitting results, and additional graphs) is available online. This material is available free of charge via the Internet at http://pubs.acs.org.

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